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On the direct CP-violation parameter ϵ'

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Abstract. We review different definitions of the ϵ' parameter describing direct CP-violation in neutral kaon decays, which was precisely measured in recent experiments, and point out the inconsistency of some of them due to a CPT-constraint. The proper comparison of the experimental results to the theoretical computations is discussed.

1 Introduction

Among the important achievements of experimental physics in the past few years, the clarification of the long-standing puzzle concerning the existence of direct CP-violation in nature has an important place. The definitive proof that CP-violation is indeed present in the decay amplitudes of the long-lived neutral kaon to $\pi\pi$ final states [1, 2], as expressed by the small but non-zero parameter ϵ', is the culmination of an experimental program which started 40 years ago, right after the discovery of CP-violation [3], and was strongly pursued since then, with several dedicated efforts in the past two decades (see e.g. [7] for a recent review).

The deep meaning of such a result lies in the indication that CP-violation, being present also in its direct form as expected from the current CKM paradigm, is truly an ubiquitous feature of weak interactions, not limited to the peculiar K^0–$\overline{K^0}$ system as the super-weak ansatz [6] would suggest. This fact was experimentally confirmed just a few years after the definitive proof of direct CP-violation, when CP-violation in the neutral B meson system was measured with significant statistics at the B-factories [4, 5].

While the main importance of the result is expressed by the fact that $\epsilon' \neq 0$ (with a significance which at present exceeds 7 standard deviations), regardless of its exact value, one should not oversee the fact that this parameter is now measured at the $\sim 15\%$ level, and improvements on the precision are expected when the final result from the full KTeV statistics and data from KLOE will be available.

Although the theoretical control of the ϵ' parameter is still poor at present, the situation is expected to improve in the future, particularly due to progress in lattice QCD computations, and the CP-violating parameters of the K meson system could also acquire more value as *quantitative* tests of the standard model as well as constraints on models of new physics.

In this perspective, it seems appropriate to establish a clean framework in which experimental measurements are to be compared with the theoretical predictions and among themselves, while avoiding possible confusion which could arise due to the existence of several alternative formulations of the phenomenological description.

The plan of this paper is as follows: in Sect. 2 we briefly review a simple and consistent parameterization of CP-violation in the K system, and in Sect. 3 we compare it with other formulations appearing in the literature, pointing out in Sect. 4 some inconsistencies which are usually overlooked. We then summarize in Sect. 5 the experimental knowledge on the ϵ' parameter. Finally, Sect. 6 presents our conclusions.

2 CP-violating parameters in the neutral K system

The phenomenological description of CP-violation in the neutral kaon system has its roots in the classic seminal papers by Wu, Yang and Lee [8, 9]. Such a description involves the two complex parameters ϵ and ϵ', intended to parameterize respectively the so-called "indirect" CP-violation, defined [11] as the one occurring in the $|\Delta S| = 2$ virtual transitions described by the effective Hamiltonian in the K^0–$\overline{K^0}$ sub-space, and the "direct" CP-violation occurring in the physical $|\Delta S| = 1$ decay amplitudes to real final states such as $\pi^+\pi^-$ or $\pi^0\pi^0$.

In discussing CP-violation, care should be taken in considering which parameters are unphysical because their value depends on the arbitrary choice of the phase for the state vectors representing the different particles; indeed, there is a considerable amount of literature concerning the proper definition of rephasing-invariant parameters in the kaon system (see e.g. [12–14]).

We now introduce some definitions [10, 15]; we will assume the validity of CPT-symmetry in the following, unless explicitly indicated otherwise.

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The CP-violating measurable ratios of amplitudes for decays of neutral kaons into a final CP-eigenstate $|f\rangle$ with eigenvalue $CP=+1$ are

$$\eta_f \doteq \frac{\langle f|\mathcal{T}|K_\mathrm{L}\rangle}{\langle f|\mathcal{T}|K_\mathrm{S}\rangle}\frac{\langle K^0|K_\mathrm{S}\rangle}{\langle K^0|K_\mathrm{L}\rangle}, \tag{1}$$

where $\mathcal{T}$ is the transition matrix of weak interactions, and the second factor makes the η_f parameter invariant under rephasing of both the $|K^0\rangle, |\overline{K^0}\rangle$ and $|K_\mathrm{S}\rangle, |K_\mathrm{L}\rangle$ state vectors [16]; such a factor is often omitted, implicitly making the choice of a phase convention in which its value is 1.

In an analog way, rephasing-invariant amplitude ratios can be defined for other (non-observable) CP-even final states, such as those with two pions in a definite isospin eigenstate with eigenvalue I:

$$\eta_I \doteq \frac{\langle(\pi\pi)_I|\mathcal{T}|K_\mathrm{L}\rangle}{\langle(\pi\pi)_I|\mathcal{T}|K_\mathrm{S}\rangle}\frac{\langle K^0|K_\mathrm{S}\rangle}{\langle K^0|K_\mathrm{L}\rangle}, \tag{2}$$

and the usual ϵ parameter is defined as

$$\epsilon \doteq \eta_0. \tag{3}$$

The quantity

$$\omega \doteq \frac{\langle(\pi\pi)_{I=2}|\mathcal{T}|K_\mathrm{S}\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{T}|K_\mathrm{S}\rangle} \tag{4}$$

parameterizes the violation of the so-called (and poorly understood) $\Delta I=1/2$ rule. The modulus of such a parameter can be extracted from the $\pi\pi$ decay rates of K^+ (which require $\Delta I>1/2$) and K_S (see e.g. [15]) and is $|\omega|\simeq 0.045$.

The direct CP-violation parameter is finally

$$\begin{aligned}\epsilon' \doteq{}& \frac{1}{\sqrt{2}}\left[\frac{\langle(\pi\pi)_{I=2}|\mathcal{T}|K_\mathrm{L}\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{T}|K_\mathrm{S}\rangle}\right.\\ &\left.-\frac{\langle(\pi\pi)_{I=0}|\mathcal{T}|K_\mathrm{L}\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{T}|K_\mathrm{S}\rangle}\frac{\langle(\pi\pi)_{I=2}|\mathcal{T}|K_\mathrm{S}\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{T}|K_\mathrm{S}\rangle}\right]\frac{\langle K^0|K_\mathrm{S}\rangle}{\langle K^0|K_\mathrm{L}\rangle}\\ ={}& \frac{\omega}{\sqrt{2}}(\eta_2-\eta_0).\end{aligned} \tag{5}$$

All the above parameters are explicitly invariant under rephasing of both $|K^0\rangle, |\overline{K^0}\rangle$ and $|K_\mathrm{S}\rangle, |K_\mathrm{L}\rangle$ and agree with those used in a significant number of papers describing the phenomenology of CP-violation in neutral kaon decays.

CPT-symmetry and the symmetry of strong interactions under time reversal (plus unitarity) allow us to write the decay amplitudes for $K^0, \overline{K^0}$ into $\pi\pi$ states of definite isospin as

$$\begin{aligned}A_I &\doteq \langle(\pi\pi)_I|\mathcal{T}|K^0\rangle = a_I \mathrm{e}^{\mathrm{i}\delta_I},\\ \overline{A}_I &\doteq \langle(\pi\pi)_I|\mathcal{T}|\overline{K^0}\rangle = a_I^* \mathrm{e}^{\mathrm{i}\delta_I},\end{aligned} \tag{6}$$

factorizing the CP-even $\pi\pi$ scattering phase δ_I by the Fermi–Watson theorem.

It can be easily seen [11] that the parameter ϵ contains both indirect and direct CP-violation, and the separation of the two parts has no physical meaning, depending on the choice of phase convention:

$$\epsilon = \frac{\overline{\epsilon}+\mathrm{i}\xi_0}{1+\mathrm{i}\overline{\epsilon}\,\xi_0}, \tag{7}$$

where $\overline{\epsilon}$ is the (phase-convention dependent) mixing parameter which describes the $K^0, \overline{K^0}$ components in the physical eigenstates of the effective Hamiltonian contributing to indirect CP-violation:

$$|K_\mathrm{S}\rangle = \tfrac{1}{\sqrt{2(1+|\overline{\epsilon}|^2)}}\left[(1+\overline{\epsilon})|K^0\rangle + (1-\overline{\epsilon})|\overline{K^0}\rangle\right], \tag{8}$$

$$|K_\mathrm{L}\rangle = \tfrac{1}{\sqrt{2(1+|\overline{\epsilon}|^2)}}\left[(1+\overline{\epsilon})|K^0\rangle - (1-\overline{\epsilon})|\overline{K^0}\rangle\right] \tag{9}$$

(having arbitrarily fixed the relative phase between $|K_\mathrm{S}\rangle$ and $|K_\mathrm{L}\rangle$), and $\xi_I \doteq \mathrm{Im}(a_I)/\mathrm{Re}(a_I)$ is a measure of the (unphysical) weak phase of the decay amplitude of K^0 into a $\pi\pi$ state of isospin I. It should be recalled that the unphysical parameter $\overline{\epsilon}$ can be very large even if CP-violation itself is a small effect, i.e. one can choose a phase convention in which $|\overline{\epsilon}|\sim 10^3$ (see e.g. [12] and references therein).

On the contrary, the parameter ϵ' is entirely due to direct CP-violation, as can be seen explicitly by rewriting it as

$$\epsilon' = \frac{\mathrm{i}}{\sqrt{2}}\omega(1-\overline{\epsilon}^2)\frac{\xi_2-\xi_0}{(1+\mathrm{i}\overline{\epsilon}\,\xi_0)(1+\mathrm{i}\overline{\epsilon}\,\xi_2)}, \tag{10}$$

which makes evident that a (phase) difference of the weak decay amplitudes to two isospin channels is required to have $\epsilon'\neq 0$ (we remark in passing that a difference in the strong phases δ_I is not required to have $\epsilon'\neq 0$, since this parameter also gets a contribution from the interference of decays with and without mixing [7]).

In terms of the above parameters, the amplitude ratios for $\pi^+\pi^-$ and $\pi^0\pi^0$ decays are written respectively as

$$\eta_{+-} = \epsilon + \frac{\epsilon'}{1+\omega/\sqrt{2}}, \quad \eta_{00} = \epsilon - \frac{2\epsilon'}{1-\omega\sqrt{2}}. \tag{11}$$

3 Alternative formulations

Several other definitions of the CP-violating parameters for the neutral K system are used in the past and recent literature: most of those coincide when suitable approximations are done; some of these approximations are physically justified in terms of small parameters, while others just depend on arbitrary phase-convention choices.

Ignoring definitions which differ from the one described above just for trivial factors of $\sqrt{2}$, the most frequent cases found in the literature are the following[1].

[1] In listing the alternative definitions we always use the same symbols $\tilde{\epsilon}, \tilde{\omega}, \tilde{\epsilon}'$ in order to distinguish them from our standard definitions in (3), (4) and (5), but they clearly refer to different quantities in each case.

(1) A minor variation [23] is that in which only the definition of ϵ' is changed to

$$\tilde{\epsilon}' \doteq \frac{\omega}{\sqrt{2}}\left(\mathrm{i}\,\xi_2 + \overline{\epsilon}\right), \qquad (12)$$

so that

$$\eta_{+-} = \frac{\epsilon + \tilde{\epsilon}'}{1 + \omega/\sqrt{2}}, \;\; \eta_{00} = \frac{\epsilon - 2\,\tilde{\epsilon}'}{1 - \omega\sqrt{2}}. \qquad (13)$$

(2) Another definition [21] is

$$\tilde{\epsilon}' \doteq \frac{1}{\sqrt{2}}\frac{\langle(\pi\pi)_{I=2}|\mathcal{T}|K_{\mathrm{L}}\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{T}|K_{\mathrm{S}}\rangle} - \frac{\mathrm{i}}{\sqrt{2}}\,\xi_0\,\omega, \qquad (14)$$

to which the definition in (5) reduces in the limit $|\overline{\epsilon}| \ll 1$.
(3) Some authors [17] define the parameters in terms of the weak phases of $K^0, \overline{K^0}$ decay amplitudes into isospin eigenstates, i.e.

$$\tilde{\epsilon} \doteq \overline{\epsilon} + \mathrm{i}\,\xi_0, \qquad (15)$$

$$\tilde{\omega} \doteq \frac{\mathrm{Re}(a_2)}{\mathrm{Re}(a_0)}\mathrm{e}^{\mathrm{i}(\delta_2-\delta_0)}, \qquad (16)$$

$$\tilde{\epsilon}' \doteq \frac{\mathrm{i}}{\sqrt{2}}\omega(\xi_2 - \xi_0). \qquad (17)$$

The expressions for η_{+-} and η_{00} read in this case

$$\eta_{+-} = \tilde{\epsilon} + \frac{\tilde{\epsilon}' - \mathrm{i}\overline{\epsilon}\,\tilde{\epsilon}\,(\xi_0 + \xi_2\,\tilde{\omega}/\sqrt{2})}{1 + \tilde{\omega}/\sqrt{2} + \mathrm{i}\overline{\epsilon}\,(\xi_0 + \xi_2\,\tilde{\omega}/\sqrt{2})}, \qquad (18)$$

$$\eta_{00} = \tilde{\epsilon} - \frac{2\tilde{\epsilon}' - \mathrm{i}\overline{\epsilon}\,\tilde{\epsilon}\,(\xi_0 - \xi_2\,\tilde{\omega}\sqrt{2})}{1 - \tilde{\omega}/\sqrt{2} + \mathrm{i}\overline{\epsilon}\,(\xi_0 - \xi_2\,\tilde{\omega}\,\sqrt{2})}, \qquad (19)$$

and reduce to the ones in (11) when terms of order $\omega\,\xi_I$ and $\omega^2\,\epsilon'/\epsilon$ are neglected.
(4) Another widespread definition [18–20] is the one in terms of the ratios of amplitudes for $K_{\mathrm{S}}, K_{\mathrm{L}}$ decays into physical states:

$$\tilde{\epsilon} \doteq (2\eta_{+-} + \eta_{00})/3, \qquad (20)$$

$$\tilde{\epsilon}' \doteq (\eta_{+-} - \eta_{00})/3, \qquad (21)$$

which is obviously tailored to get exactly

$$\eta_{+-} = \tilde{\epsilon} + \tilde{\epsilon}' \qquad \eta_{00} = \tilde{\epsilon} - 2\,\tilde{\epsilon}'. \qquad (22)$$

The definitions based on (22) are sometimes described as the "experimental" ones for the ϵ and ϵ' parameters.
(5) Other definitions [24] are based on the decay amplitudes of the CP-eigenstates K_1 ($CP = +1$) and K_2 ($CP = -1$):

$$\tilde{\epsilon}' \doteq \frac{1}{\sqrt{2}}\frac{\langle(\pi\pi)_{I=2}|\mathcal{T}|K_2\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{T}|K_1\rangle} = \frac{\omega}{\sqrt{2}}\frac{\eta_2 - \overline{\epsilon}}{1 - \overline{\epsilon}\,\eta_0}. \qquad (23)$$

This definition makes very explicit the meaning of $\tilde{\epsilon}'$ as a direct CP-violation parameter, but its relations with the observable quantities become more complicated. $\tilde{\epsilon}'$ in (23) reduces to (5) in the limit in which CP-violation is small and $|\overline{\epsilon}| \ll 1$; in this case the phase of $\tilde{\epsilon}'$ (assuming CPT) is exactly $\delta_2 - \delta_0 + \pi/2$, and (13) are valid.
(6) The previous scheme can be generalized [26] introducing for each final state f the quantities

$$\tilde{\epsilon}'_f \doteq \frac{1 - \overline{A}_f/A_f}{1 + \overline{A}_f/A_f} = \frac{\overline{\epsilon} - \eta_f}{1 - \overline{\epsilon}\,\eta_f}, \qquad (24)$$

in analogy to the expression

$$\overline{\epsilon} = \frac{1 - q/p}{1 + q/p}, \qquad (25)$$

where

$$p \doteq \langle K^0|K_{\mathrm{S}}\rangle = (1 + \overline{\epsilon}),$$
$$q \doteq \langle\overline{K^0}|K_{\mathrm{S}}\rangle = (1 - \overline{\epsilon}). \qquad (26)$$

In any phase convention in which $|\overline{\epsilon}| \ll 1$, the expression for η_f reduces in this case to

$$\eta_f \simeq \overline{\epsilon} + \tilde{\epsilon}'_f, \qquad (27)$$

and for the $\pi\pi$ states, in the limit $|\omega| \ll 1$,

$$\tilde{\epsilon}'_{+-} \simeq \mathrm{i}\,\xi_0 + \epsilon', \;\; \tilde{\epsilon}'_{00} \simeq \mathrm{i}\,\xi_0 - 2\,\epsilon', \qquad (28)$$

giving back (22).

We remind the reader that while both $\overline{\epsilon}$ and the quantity in (24) are not rephasing invariant, the quantity

$$\lambda_f \doteq \frac{1 - q\overline{A}_f/pA_f}{1 + q\overline{A}_f/pA_f}, \qquad (29)$$

commonly used in the phenomenological description of CP-violation in the B system, has this property. In terms of such a parameter the quantities defined in (3) and (5) are

$$\epsilon = \frac{1 - \lambda_0}{1 + \lambda_0}, \;\; \epsilon' = \sqrt{2}\,\mathrm{e}^{\mathrm{i}(\delta_2-\delta_0)}\frac{a_2}{a_0}\frac{\lambda_0 - \lambda_2}{(1 + \lambda_0)^2}, \qquad (30)$$

while the expressions for the quantities (20) and (21) are more complicated and not very illuminating.

The actual direct CP-violation parameter which theorists have been trying to compute for a long time with different approaches (see e.g. [30] for a recent review on the theoretical status of ϵ' computations) is expressed by [22]

$$\epsilon'_{TH} \doteq \frac{\mathrm{i}}{\sqrt{2}}e^{\mathrm{i}(\delta_2-\delta_0)}\frac{\mathrm{Im}(a_0)}{\mathrm{Re}(a_0)}\left(\frac{\mathrm{Im}(a_2)}{\mathrm{Im}(a_0)} - \frac{\mathrm{Re}(a_2)}{\mathrm{Re}(a_0)}\right). \qquad (31)$$

Indeed, as CP-violation is small, the definition in (5) reduces to (31) in any phase convention in which $|\overline{\epsilon}| \ll 1$ neglecting terms of order $|\overline{\epsilon}\,\xi_I|$, when

$$\omega \simeq \mathrm{e}^{\mathrm{i}(\delta_2-\delta_0)}\frac{\mathrm{Re}(a_2)}{\mathrm{Re}(a_0)}, \qquad (32)$$

$$\eta_I \simeq \overline{\epsilon} + \mathrm{i}\,\xi_I, \qquad (33)$$

$$\epsilon' \simeq \frac{\mathrm{i}}{\sqrt{2}} \mathrm{e}^{\mathrm{i}(\delta_2-\delta_0)} \frac{\mathrm{Re}(a_2)}{\mathrm{Re}(a_0)} (\xi_2 - \xi_0), \tag{34}$$

without any approximation based on the size of $|\omega|$ (in (34) $|\xi_I| \ll 1$ was also assumed).

When neglecting $|\omega|$ and adopting the phase convention in which the dominant amplitude a_0 is real (the so-called Wu–Yang phase convention), one recovers the original expression of [8].

The PDG review on CP-violation [27] adopts the definitions of (22) with (17), noting that one obtains (31) when terms of order $\epsilon' \mathrm{Re}(a_2/a_0)$ are neglected.

4 Consistency requirements

We would like to point out that not all the above definitions are consistent. In particular the so-called "experimental" expressions in (22) can only be considered as the approximations of the exact (11) for $|\omega| \ll 1$, and cannot be promoted to alternative *definitions* of the ϵ and ϵ' parameters.

The reason is that there is an additional constraint that the amplitudes should satisfy, dictated by CPT-symmetry (which we have assumed throughout). Ignoring electromagnetic effects (consistent with our neglecting of isospin-breaking effects), the $\pi\pi$ final states are not connected by strong interactions to other states: the 3π states for zero total angular momentum have opposite parity (conserved by strong interactions), and the $\pi\pi\gamma$ states require electromagnetism. It follows that CPT-symmetry by itself requires the equality of partial decay rates for particle and antiparticle:

$$\begin{aligned} &\Gamma(K^0 \to \pi^+\pi^-) + \Gamma(K^0 \to \pi^0\pi^0)\,, \\ &= \Gamma(\overline{K^0} \to \pi^+\pi^-) + \Gamma(\overline{K^0} \to \pi^0\pi^0). \end{aligned} \tag{35}$$

This constraint can be expressed as a function of the physical decay amplitudes for $K_\mathrm{S}, K_\mathrm{L}$ and the mixing parameters:

$$\begin{aligned} &|\langle \pi^+\pi^-|\mathcal{T}|K_\mathrm{S}\rangle|^2 \left[2\mathrm{Re}(\eta_{+-}) - \langle K_\mathrm{S}|K_\mathrm{L}\rangle(1+|\eta_{+-}|^2)\right] \\ &\quad + |\langle \pi^0\pi^0|\mathcal{T}|K_\mathrm{S}\rangle|^2 \left[2\mathrm{Re}(\eta_{00}) - \langle K_\mathrm{S}|K_\mathrm{L}\rangle(1+|\eta_{00}|^2)\right] \\ &= 0, \end{aligned} \tag{36}$$

which is explicitly invariant for rephasing of the $|K^0\rangle, |\overline{K^0}\rangle$ states, since

$$\langle K_\mathrm{S}|K_\mathrm{L}\rangle = \frac{2\mathrm{Re}(\overline{\epsilon})}{1+|\overline{\epsilon}|^2} = \frac{2\mathrm{Re}(\epsilon)}{1+|\epsilon|^2}. \tag{37}$$

Writing, without any loss of generality,

$$\eta_{+-} = \epsilon + \epsilon_{+-}\,, \qquad \eta_{00} = \epsilon + \epsilon_{00}, \tag{38}$$

and using the isospin decomposition of the decay amplitudes (neglecting $|\Delta I| > 3/2$ amplitudes and isospin-breaking effects), the constraint equation reduces to

$$\begin{aligned} &|1+\omega/\sqrt{2}|^2 \left[2\,\mathrm{Re}(\epsilon_{+-}) - \langle K_\mathrm{S}|K_\mathrm{L}\rangle|\epsilon_{+-}|^2 \right. \\ &\quad \left. -2\,|\epsilon|^2 \langle K_\mathrm{S}|K_\mathrm{L}\rangle\, \mathrm{Re}(\epsilon_{+-}/\epsilon)\right] \\ &+ |1/\sqrt{2}-\omega|^2 \left[2\,\mathrm{Re}(\epsilon_{00}) - \langle K_\mathrm{S}|K_\mathrm{L}\rangle|\epsilon_{00}|^2 \right. \\ &\quad \left. -2\,|\epsilon|^2 \langle K_\mathrm{S}|K_\mathrm{L}\rangle\, \mathrm{Re}(\epsilon_{00}/\epsilon)\right] = 0. \end{aligned} \tag{39}$$

Since we know experimentally that $|\epsilon| = O(10^{-3})$ and $|\epsilon_{+-}|, |\epsilon_{00}| = O(10^{-6})$, we keep terms up to first order in $|\epsilon_{+-}|$ and $|\epsilon_{00}|$, obtaining

$$|1+\omega/\sqrt{2}|^2\, \mathrm{Re}(\epsilon_{+-}) + |1/\sqrt{2}-\omega|^2\, \mathrm{Re}(\epsilon_{00}) = 0. \tag{40}$$

Now, keeping only terms which are first order in $|\omega|$ and using the experimental fact that $\delta_2 - \delta_0 \simeq -\pi/4$, one is finally led to

$$2\,(1+|\omega|)\, \mathrm{Re}(\epsilon_{+-}) + (1-2|\omega|)\, \mathrm{Re}(\epsilon_{00}) = 0. \tag{41}$$

It seems that such a constraint was not discussed in this context in the literature.

Clearly, this equation is trivially satisfied in the absence of direct CP-violation, when $\epsilon_{+-} = \epsilon_{00} = 0$.

In the stronger approximation in which all terms containing $|\omega|$ are neglected, the constraint becomes

$$2\,\mathrm{Re}(\epsilon_{+-}) = -\mathrm{Re}(\epsilon_{00}), \tag{42}$$

which is satisfied by the choice

$$\epsilon_{+-} = \epsilon', \qquad \epsilon_{00} = -2\,\epsilon', \tag{43}$$

so that one gets back the approximate (22).

Using instead the relations in (11), consistent with the definitions in (3), (4) and (5), one has

$$\epsilon_{+-} = \frac{\epsilon'}{1+\omega/\sqrt{2}}, \qquad \epsilon_{00} = \frac{-2\,\epsilon'}{1-\omega\sqrt{2}}, \tag{44}$$

for which the constraint of (40) is equivalent to

$$\mathrm{Re}(\epsilon'\omega^*) = 0, \tag{45}$$

which is indeed satisfied at the level of approximation considered here: writing the phase of ϵ' (defined in (5)) in terms of the phase of ω as

$$\phi(\epsilon') = \phi(\omega) + \pi/2 + \delta\phi, \tag{46}$$

the constraint (45) requires $\delta\phi = 0 \pmod{\pi}$. Since the exact expression for ϵ' is

$$\epsilon' = \frac{\mathrm{i}\omega}{\sqrt{2}} \left[(\xi_2 - \xi_0) \frac{1-\overline{\epsilon}^2}{(1+\mathrm{i}\,\overline{\epsilon}\,\xi_0)(1+\mathrm{i}\,\overline{\epsilon}\,\xi_2)} \right]. \tag{47}$$

$\delta\phi$ is the phase of the term in square brackets in the above expression (47), which can be seen to be indeed small by using the phase convention $|\overline{\epsilon}| \ll 1$, since it is an invariant quantity under rephasing as can be easily verified. Its value is $\delta\phi \simeq 0.6 \cdot 10^{-5}$.

It should be mentioned that a phase space correction factor is required to account for the difference in the $\pi^\pm$

and π^0 masses when expressing the partial decay rates in terms of the amplitudes:

$$a_{\rm PS} = \frac{\sqrt{m(K^0)^2 - 4m(\pi^\pm)^2}}{\sqrt{m(K^0)^2 - 4m(\pi^0)^2}} \simeq 0.9855. \qquad (48)$$

Strictly speaking, this 1.5% effect should be neglected consistently in the exact isospin limit; partially accounting for isospin breaking in this way, the constraint equation (40) is modified into

$$a_{\rm PS}|1+\omega/\sqrt{2}|^2\,{\rm Re}(\epsilon_{+-}) + |1/\sqrt{2} - \omega|^2\,{\rm Re}(\epsilon_{00}) = 0. \quad (49)$$

The expressions in (11) still satisfy this constraint at the same level of approximation as before: in this case instead of (45) one gets

$$(a_{\rm PS}-1)\,{\rm Re}(\epsilon') + \frac{2+a_{\rm PS}}{\sqrt{2}}\,{\rm Re}(\epsilon'\omega^*) = 0, \qquad (50)$$

but the left-hand side of (50) can be seen to be still proportional to $\delta\phi \simeq 0$.

Summarizing, the expressions in (22) are approximations which are valid in the limit in which the parameter $|\omega|$ parameterizing the violation of the $\Delta I = 1/2$ rule is neglected, and cannot be considered as consistent alternative definitions of parameters describing CP-violation in the kaon system. It should be noted, furthermore, that the approximation in which $|\omega|$ is neglected is – strictly speaking – not a consistent one in this context, since if $\omega = 0$ the absence of $\Delta I = 3/2$ amplitudes would imply that no direct CP-violation is possible for neutral kaons decays into $\pi\pi$, due to lack of an amplitude interfering with the dominant ($\Delta I = 1/2$) one.

5 Experimental results

In the experiments performed so far with neutral K mesons, the information on direct CP-violation is extracted from the experimental measurement of the so-called “double ratio” R of partial decay widths:

$$R \doteq \frac{\Gamma(K_{\rm L}\to\pi^0\pi^0)}{\Gamma(K_{\rm S}\to\pi^0\pi^0)}\,\frac{\Gamma(K_{\rm S}\to\pi^+\pi^-)}{\Gamma(K_{\rm L}\to\pi^+\pi^-)} = \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2. \quad (51)$$

This quantity is related to ϵ'/ϵ by the following approximate expression:

$$R \simeq 1 - 6{\rm Re}(\epsilon'/\epsilon) - 3\sqrt{2}\,{\rm Re}\left(\omega^*\,\epsilon'/\epsilon\right), \qquad (52)$$

in which second order terms in ϵ' or ω were neglected. It is well known that the ϵ'/ϵ ratio is close to being real, since [2, 28, 31]

$$\phi(\epsilon) \simeq 2\Delta m/\Delta\Gamma = (43.46 \pm 0.05)^\circ, \qquad (53)$$

$$\phi(\epsilon') \simeq \delta_2 - \delta_0 + \pi/2 = (48 \pm 4)^\circ \qquad (54)$$

(here, as usual, $\Delta m \doteq m(K_{\rm L}) - m(K_{\rm S}) > 0$, $\Delta\Gamma \doteq \Gamma_S - \Gamma_L > 0$), where the first approximate equality becomes exact in the limit in which the $\pi\pi$ decay amplitude dominates (always assuming CPT-symmetry), while the second one only depends on the smallness of CP-violation. Equation (52) therefore reduces to [32]

$$R \simeq 1 - 6\,{\rm Re}(\epsilon'/\epsilon)\left[1 + {\rm Re}(\omega)/\sqrt{2}\right], \qquad (55)$$

which is commonly approximated to

$$R \approx 1 - 6\,{\rm Re}(\epsilon'/\epsilon), \qquad (56)$$

by neglecting $|\omega|$. Equation (56) is the one routinely used in experimental papers.

Since $|\omega|$ is of order 5%, the difference between using (55) and (56) amounts to a reduction of the value of ${\rm Re}(\epsilon'/\epsilon)$ by 2.2%, which is small when compared to the current precision of the theoretical computations, and also to the present experimental error (but not to the size of the systematic corrections applied by the experiments to obtain the central value). Extrapolating to a final experimental precision of $1\cdot 10^{-4}$ on ${\rm Re}(\epsilon'/\epsilon)$ in a few years from now, the use of the correct expression (55) will be appropriate.

Averaging the most precise results on ${\rm Re}(\epsilon'/\epsilon)$ at face value one obtains ${\rm Re}(\epsilon'/\epsilon) = (16.7 \pm 2.3)\cdot 10^{-4}$ where the error has been inflated by a factor 1.44 according to the procedure adopted by the PDG [28], due to the poor χ^2 value of 6.2 (with 3 degrees of freedom).

Using (55) one obtains instead

$${\rm Re}(\epsilon'/\epsilon) = (16.3 \pm 2.3)\cdot 10^{-4} \qquad (57)$$

as the value to be compared to theoretical computations, and the χ^2 improves only marginally to 5.9, without affecting the scaled error in a significant way. A graphical depiction of the present data is shown in Fig. 1. The probability of the four most precise measurements to be consistent is 11%, varying between 9% and 22% when a single measurement is ignored.

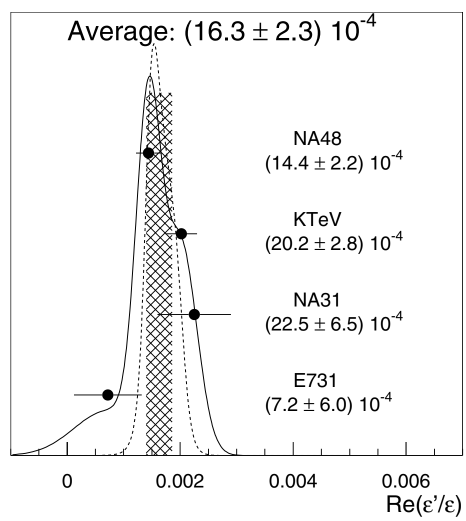

Fig. 1. Ideogram of recent published ${\rm Re}(\epsilon'/\epsilon)$ measurements as corrected according to (55). The curves show (unnormalized) probability distributions according to the PDG procedure [28] (solid line) or a Bayesian “skeptical” approach [29] (dashed line)

It should also be noted that the uncertainty on the value of $|\omega|$ hardly affects any comparison with theory in itself, since in any case the empirical value of such a parameter is used both in the computation (due to the theoretical difficulties with the $\Delta I = 1/2$ rule) and in extracting the value of $\mathrm{Re}(\epsilon'/\epsilon)$ from the experiments.

The similarity of the phase of ϵ' with that of ϵ is an accidental fact which hinges on the validity of the CPT-symmetry, without which the phase of ϵ would be different from the "super-weak" value $2\,\Delta m/\Delta\Gamma$; for this reason the smallness of $\mathrm{Im}(\epsilon'/\epsilon)$ is considered a test of such a symmetry. The difference of such phases is however experimentally constrained [2] to be tiny: $\phi(\epsilon') - \phi(\epsilon) = (-1.2 \pm 1.5)^\circ$, and therefore the use of (52) is not required; experiments usually assume CPT-symmetry explicitly [2] or implicitly [1] in the extraction of ϵ'/ϵ.

It should be recalled that $\mathrm{Im}(\epsilon'/\epsilon)$ can be measured using kaon interferometry [19, 33] and would be therefore accessible to the KLOE experiment [34] when sufficient statistics will be accumulated.

6 Conclusions

In view of the recent and future progress, in both experiment and theory, in the determination of the parameter ϵ' measuring direct CP-violation in neutral kaon decays, the use of a common definition for it is advisable. We reviewed some of the choices present in the literature, showing that the simple so-called "experimental" one is necessarily an approximation, which is still good at the present level of accuracy but would have to be abandoned in the future to allow for an accurate comparison of theory and experiment.

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